Fractal just infinite nil Lie superalgebra of finite width

Victor Petrogradsky, UnB

joint work with Otto Augusto de Morais Costa

The Grigorchuk and Gupta-Sidki groups play fundamental role in modern group theory. Their natural analogues are self-similar nil Lie p-algebras. In characteristic zero, similar examples of Lie algebras do not exist (Martinez and Zelmanov). The author recently constructed a 3-generated self-similar nil finely graded Lie superalgebra, which showed that an extension of Martinez-Zelmanov's result for Lie superalgebras of characteristic zero is not valid.

Now, we suggest a more handy example. We construct a 2-generated self-similar Lie superalgebra $\mathbf{R}$ over arbitrary field. It has a clear monomial basis, unlike many examples studied before, we find a clear monomial basis of its associative hull $\mathbf{A}$, the latter has a quadratic growth. The algebras $\mathbf{R}$ and $\mathbf{A}$ are $\mathbb{Z}^2$-graded by multidegree in generators, positions of their $\mathbb{Z}^2$-components are bounded by pairs of logarithmic curves on plane. The $\mathbb{Z}^2$-components of $\mathbf{R}$ are at most one-dimensional, thus, the $\mathbb{Z}^2$-grading of $\mathbf{R}$ is fine. As an analogue of periodicity, we establish that homogeneous elements of the grading $\mathbf{R}{=}\mathbf{R}_{\bar{0}} \oplus \mathbf{R}_{\bar{1}}$ are ad-nilpotent. In case of $\mathbb{N}$-graded algebras, a close analogue to being simple is being just-infinite. We prove that $\mathbf{R}$ is just infinite, but not hereditary just infinite. Our example is close to a smallest possible example, because $\mathbf{R}$ has a linear growth with a growth function $\gamma_{\mathbf{R}}(m) \approx 3m$, $m \to \infty$. Moreover, its degree $\mathbb{N}$-gradation is of width 4 ($\operatorname{char} K \neq 2$). In case $\operatorname{char} K = 2$, we obtain a Lie algebra of width 2 that is not thin. Our example shows again that an extension of the result of Martinez and Zelmanov for Lie superalgebras of characteristic zero is not valid.