Graded algebras with σ-involutions.

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Let R be a commutative associative unitary ring. We will consider associative non-commutative R-algebras graded by a finite abelian group G. Consider a skewsymmetric 2-cocycle $\sigma : G \times G \to U(R)$ stisfying the condition $\sigma(\alpha, -\alpha)^2 = 1$ for all $\alpha \in G$. A σ-involution is a graded R-linear map of the 2-nd order $*_\sigma : A \to A$, defined in a G-graded R-algebra A, which satisfies the condition

$$(a_\alpha b_\beta)^{*_\sigma} = \sigma(\alpha, \beta) \;\; b_\beta^{*_\sigma} a_\alpha^{*_\sigma}, \quad \forall a_\alpha \in A_\alpha, b_\beta \in A_{beta}, \quad \alpha, \beta \in G.$$

Observe that a σ-involution is a natural generalization of notions of a graded involution and a superinvolution of a superalgebra.

We will consider bilinear and sesqilinear graded forms and pairings of associative graded R-algebras and discuss some generalizations of various classical results for this case. Particularly, we will present analogues of Riesz representation theorem and Kaplansky's theorem [1] for σ-involutions of associative graded R-algebras. The last theorem shows the relations of σ-involutions with sesqilinear graded forms in primitive graded algebras, and implies a description of finite dimensional $*_\sigma$-simple graded algebras.

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References

[1] N. Jacobson, Structure of Rings, AMS Colloquium Publication 37, AMS, Providence, R.I., 1964.

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