On a problem of N.Jacobson

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The Jacobson Coordinatization Theorem for Cayley algebras [1] states that if a unital alternative algebra A contains a Cayley algebra C containing 1 then $A \cong B \otimes C$ for some associative and commutative algebra B. In the same paper Jacobson formulated a question on the structure of alternative algebras that contain a quaternion subalgebra with the same unit:

"In the alternative case one can prove, using Zorn's methods, that if $\mathfrak{B}$ is alternative with an identity and $\mathfrak{A}$ is an associative central simple subalgebra containing 1 and of more than 4 dimensions, then $\mathfrak{B}$ is necessarily associative. In this case Wedderburn's theorem applies. The example of the Cayley algebra shows that this is not the case if $\mathfrak{A}$ is a quaternion algebra. Nevertheless, it is conceivable that the structure of alternative algebras $\mathfrak{B}$ with 1 containing a quaternion subalgebra containing 1 can be fully determined."

In our talk we will describe the structure of such algebras.

This is a joint result with Victor Hugo López Solís.

References

[1] Jacobson, N.. A Kronecker factorization theorem for Cayley algebras and the exceptional simple Jordan algebra. Amer. J. Math. 76, (1954). 447–452.