

The Okounkov-Vershik approach to the representation theory of $G \sim S_n$

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Abstract

In the Okounkov-Vershik approach to the complex irreducible representations of S_n and $G \sim S_n$ we parametrize the irreducible representations and their bases by spectral objects rather than combinatorial objects and then, at the end, give a bijection between the spectral and combinatorial objects. The fundamental ideas are similar in both cases but there are additional technicalities involved in the $G \sim S_n$ case. This was carried out by Pushkarev (*J. Math. Sci.* **96**, 3590-3599 (1999)).

The present work gives a fully detailed exposition of Pushkarev's theory incorporating the following new elements

- Our definition of a Gelfand-Tsetlin subspace, based on a multiplicity free chain of subgroups, leads to a more natural development of the theory.
- Ceccherini-Silberstein, Scarabotti, and Tolli (*Adv. Math.* **206**, 503-537 (2006)) defined the “generalized Johnson scheme”, a certain multiplicity free $G \sim S_n$ permutation module. We give an algorithm to explicitly write down the Gelfand-Tsetlin subspaces of this module. This gives the simplest nontrivial example of the Okounkov-Vershik theory.